Tone-Reservation based on fractional Fourier Transform for Chirp-based OFDM

Shi Pengfei, M. R. Anjum, Zhao Yue, Riaz Ahmed Soomro, Farhan Manzoor

Abstract—As an adaptive method to combat doubly selective channel, chirp-based orthogonal frequency-division multiplexing (Chirped-OFDM) also suffers from a high peak-to average power ratio (PAPR). In this paper, an efficient optimization for tone-reservation technique based on the structure of fractional Fourier Transform is developed to reduce the number of variables in the quadratically constrained quadratic program (QCQP) for Chirped-OFDM subcarrier modulation, which reduces complexity considerably. Simulation results yield its performance close to conventional tone reservation with less computational complexity.

Index Terms—Chirped-OFDM, fractional FFT, PAPR, tone reservation

---

1 Introduction

CHIRPED-OFDM system obtains good performance in doubly dispersive (i.e., time and frequency selective) channel[1]. However, as the multicarrier system, the relatively high PAPR is also a major drawback of Chirped–OFDM system. The problem directly influences operation cost and efficiency of the system. In order to relieve the problem of high PAPR in Chirped-OFDM system, Conventional PAPR reduction techniques in OFDM system in OFDM system are introduced, such as Clipping, Selecting Mapping(SLM)， Partial Transmit Sequence(PTS), etc[2]. Besides, an efficient tone reservation technique is proposed by Tellado in [3] to overcome those drawbacks, especially, in a large number of subcarriers. A small number of subcarriers are reserved in transmitting to reduce PAPR. However, this algorithm also suffers large of FFT/IFFT operations due to large number of iterations.

In this paper, simplification for tone-reservation technique based on the structure of fractional Fourier Transform is proposed to reduce the number of variables in the QCQP for Chirped-OFDM subcarrier modulation. The reminder of this paper is organized as follows. In section 2, we introduce the Chirped-OFDM system model and the traditional PAPR reduction schemes. Section 3 describes conventional tone reservation technique. The proposed technique based on Radix-2 FRFT algorithm and its simulations are presented in section 4.

2 Chirped OFDM System and Its PAPR

2.1 Chirped-OFDM system

The structure of Chirped- OFDM system is shown in Fig.1.

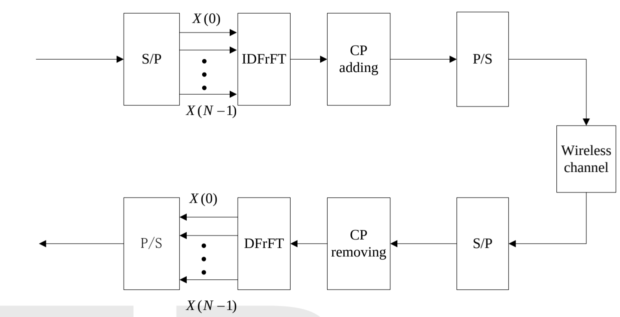

Fig. 1. Block diagram of Chirped-OFDM system

At the transmitter side, an N-point IFRFT is applied to data block symbols after transforming the high-speed data stream with digital modulated into low-speed parallel data streams. In the Chirped-OFDM system, the exponential fundamental basis waveforms are replaced by the chirp fundamental basis waveforms. The signal modulated onto subcarriers is expressed as [4]

$$x(t)=\sum_{k=0}^{N-1} X(k)\cdot c_{k,-\alpha}(t) \qquad t\in[0,T_s)$$

(1)

where $X=[X(0),X(1),\cdots,X(N-1)]$ is the transmitted data with the number of subcarriers N. $c_{k,-\alpha}(t)$ is the k^{th} chirped subcarrier function given by,

$$c_{k,\alpha}(t)=\sqrt{1-j\cot\alpha}\cdot\exp\left(j\pi[\cot\alpha\cdot t^2-2kt/T_s+\cot\alpha(k\sin\alpha/T_s)^2]\right)$$

(2)

with the Chirped-OFDM symbol period T_s and $\alpha=p\pi/2$ related to the FRFT transform order p.

Correspondingly, the normalized discrete transmitted signal modulated onto subcarriers can be expressed as:

$$x(n)=\sqrt{(1-j\cdot\cot\alpha)/N}\cdot\exp\left(-j/2\cdot\cot\alpha\cdot n^2\cdot\Delta t^2\right)$$
$$\times\sum_{k=0}^{N-1}\exp\left(-j/2\cdot\cot\alpha\cdot k^2\cdot\Delta u^2+j\cdot 2\pi\cdot n\cdot k/N\right)\cdot X(k), \quad n=0,1,\cdots,N-1$$

(3)

where Δt is the sample interval in time domain, and $\Delta\mu=2\pi\cdot|\sin\alpha|/(N\cdot\Delta t)$ denotes sample interval in fractional domain.

2.2 PAPR of Chirped OFDM system

For $x(n)$ in (3) is the summation of independently multi-carriers in the transmitter, the Chirped-OFDM symbol may

---

- *Shi Pengfei is currently pursuing Ph. D degree program in Information & Communication Engineering in Beijing Institute of Technology, China, 100081. E-mail: shipengfeibit@gmail.com*
- *M. R. Anjum is currently pursuing Ph. D degree program in School of Information & Electronics Engineering in Beijing Institute of Technolo-gy,Beijing, China, 100081. E-mail: engr.muhammadrizwan@gmail.com*

International Journal of Scientific & Engineering Research Volume 4, Issue 8, August-2013
ISSN 2229-5518

have large peak, which introduces large peak to average power ratio. Based on the traditional system, the definition of PAPR for the Chirped OFDM system is the ratio of peak power and average power expressed as:

$$PAPR(\text{dB}) = 10\log_{10}\left(\max\{|s(n)|^2\} / E\{|s(n)|^2\}\right) \tag{4}$$

PAPR performance of a multicarrier system is evaluated in terms of the complementary cumulative distribution function (CCDF) of PAPR in general. The CCDF of PAPR is defined as the probability that the PAPR of the transmitted signal exceeds a given threshold $PAPR_0$,

$$CCDF = P\{PAPR > PAPR_0\} \tag{5}$$

3 TONE RESERVATION TECHNIQUE AND THE PROPOSED

In the Chirped-OFDM system, tone reservation signal $C = [C(0), C(1), ..., C(N-1)]^T$ is added to the original signal $\mathrm{X} = [X(0), X(1), \cdots, X(N-1)]^T$, then the new transform-domain signal can be expressed as[5]:

$$x(n) = IDFRFT\{X + C\} = x(n) + c(n) \tag{6}$$

where *IDFRFT* donates to the inverse discrete fractional Fourier transform with $X + C$ satisfying the condition:

$$X + C = \begin{cases} C(k) & k \in \{i_1, i_{2,...,} i_L\} \\ X(k) & k \notin \{i_1, i_{2,...,} i_L\} \end{cases} \tag{7}$$

where L and $i_1, i_2, \cdots, i_L$ donate to the number of tone reservation, and the subset of tone reservation, respectively. In order to reduce the computation burden, $C(k)(k = i_1, i_2, ..., i_L)$ always are chosen from the collection $\{1, -1\}$ or $\{\pm 1, \pm j\}$.

Then, the optimization problem for PAPR of Chirped-OFDM system is defined as

$$\min_C \xi \tag{8}$$

subject to: $\xi = \max|x(n) + c(n)|^2$

which can be formulated as a quadratically constrained quadratic program (QCQP).

According to (3), Pei proposed the FRFT algorithm by combining FFT with chirp-rate modulation in time and frequency domain, respectively. So we could optimize the problem (8) through radix-2 FRFT algorithm, either. The principle of the proposed frame is depicted in Fig. 2.

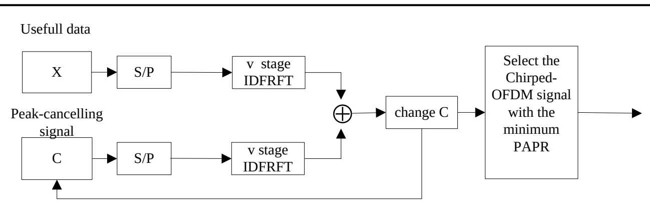

Fig. 2. The proposed tone reservation technique based on radix-2 FRFT algorithm.

Define the symbol x and index k for an intermediate stage v within the FRFT process are represented for the input X and frequency index k of each block, respectively. The new FRFT outputs on stage v:

$$x(n) = \sum_{\eta=1}^{2^{v-1}} x^{\eta}(n) = \sqrt{(1 - j\cdot\cot\alpha)/N} \times$$
$$\sum_{\eta=1}^{2^{v-1}} \sum_{k=0}^{N/2^{v-1}-1} X^{\eta}(n)\exp\left[-j/2\cdot\cot\alpha\cdot\left(n^2\cdot\Delta t^2 + k^2\cdot\Delta u^2\right)\right] W_{N/2^{v-1}}^{nk} \tag{10}$$

where W donates to DFT, which can be expressed in matrix formation:

$$\begin{bmatrix} \mathbf{x}^1 \\ \vdots \\ \mathbf{x}^{\eta} \\ \vdots \\ \mathbf{x}^{r^{v-1}} \end{bmatrix} = \begin{bmatrix} Q^1 & & & & \\ & \ddots & & & \\ & & Q^{\eta} & & \\ & & & \ddots & \\ & & & & Q^{r^{v-1}} \end{bmatrix} \begin{bmatrix} X^1 \\ \vdots \\ X^{\eta} \\ \vdots \\ X^{r^{v-1}} \end{bmatrix} \tag{11}$$

with each blocks of FRFT on stage v satisfying:

$$Q^1 = \cdots = Q^{\eta} = \cdots Q^{2^{v-1}} =$$
$$\begin{bmatrix} 1 & \cdots & 1 & \cdots & 1 \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 1 & \cdots & e^{-j/2\cdot\cot\alpha\left[n^2\cdot\Delta t^2 + k^2\cdot\Delta u^2\right]} W_{N/2^{v-1}}^{nk} & \cdots & e^{-j/2\cdot\cot\alpha\cdot\left[n^2\cdot\Delta t^2 + (N/2^{v-1}-1)^2\cdot\Delta u^2\right]} W_{N/2^{v-1}}^{n(N/2^{v-1}-1)} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ 1 & \cdots & e^{-j/2\cdot\cot\alpha\cdot\left[(N/2^{v-1}-1)^2\cdot\Delta t^2 + k^2\cdot\Delta u^2\right]} W_{N/2^{v-1}}^{(N/2^{v-1}-1)k} & \cdots & e^{-j/2\cdot\cot\alpha\left[(N/2^{v-1}-1)^2\cdot\Delta t^2 + (N/2^{v-1}-1)^2\cdot\Delta u^2\right]} W_{N/2^{v-1}}^{(N/2^{v-1}-1)^2} \end{bmatrix} \tag{12}$$

By using the intermediate stage v of FRFT algorithm, the subset Λ_e of peak reduction tones (PRT) for generating C are divided into equal subsets with length $\alpha = L/2^{v-1}$ as following, where L donates to the number of tones reservation and Λ_e is defined:

$$\Lambda_e = \left[\lambda_1^1, \cdots, \lambda_{\alpha}^1, \cdots, \lambda_1^{\eta}, \cdots, \lambda_{\alpha}^{\eta}, \cdots, \lambda_1^{r^{v-1}}, \cdots, \lambda_{\alpha}^{r^{v-1}}\right] \tag{13}$$

Then, the component $[C^1(\lambda_1^1), \cdots, C^1(\lambda_{\alpha}^1)]^T$ are randomly chosen from $\{1, -1, j, -j\}$ with subsets $\lambda_1^1, \cdots, \lambda_{\alpha}^1$ randomly from the subset of $X^1(k)$. The other C^{η} are reserved in $X^{\eta}(k)$ with the same PRT locations of $X^1(k)$. Thus, the matrices C and Q can be expressed as:

$$C = \left[C^1, \cdots, C^{\eta}, \cdots, C^{2^{v-1}}\right]^T = \left[C^1, \cdots, C^1, \cdots, C^1\right]^T \tag{14}$$

and

$$Q = \begin{bmatrix} Q^1 & & & & \\ & \ddots & & & \\ & & Q^{\eta} & & \\ & & & \ddots & \\ & & & & Q^{2^{v-1}} \end{bmatrix}_{2^{v-1}\times 2^{v-1}} = \begin{bmatrix} Q^1 & & & & \\ & \ddots & & & \\ & & Q^1 & & \\ & & & \ddots & \\ & & & & Q^1 \end{bmatrix}_{2^{v-1}\times 2^{v-1}} \tag{15}$$

Then, the optimization problem in (8) could be simplified as following:

$$\min_{C^{\eta}} \xi \tag{16}$$

subject to: $\left|x^{\eta} + Q^{\eta}C^{\eta}\right|^2 \le \tau$

Compared with (8), the proposed technique keeps the same number of constraints while reducing the number of variables from L to $L/2^{v-1}$.

International Journal of Scientific & Engineering Research Volume 4, Issue 8, August-2013
ISSN 2229-5518

4 Simulation Results

According to [6], the PAPR was extended to the case of Chirped-OFDM system, whose simulation results showed that

TABLE 1
SIMULATION PARAMETERS

Parameters	*Value*
Number of FRFT-OFDM symbols	100000
Number of subcarriers	256
Modulation mode	QPSK
Channel Type	AWGN
The value of C	$\{1,-1,j,-j\}$

the PAPR of Chirped-OFDM system was slightly relative to the fractional order p. Without loss of generality, in this section, the fractional order for the proposed technique based on radix-2 FRFT algorithm could be $p=0.1$. The other simulation parameters are in TABLE 1 and the result is presented in Fig. 3.

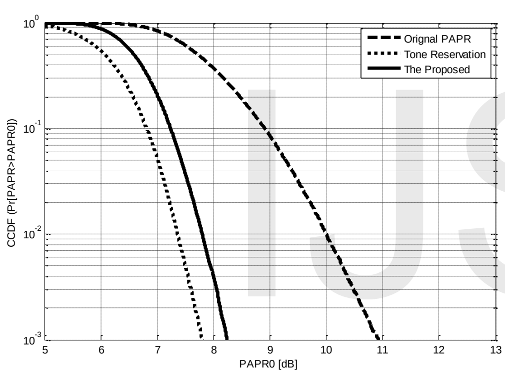

Fig. 3. PAPR comparison among traditional tone reservation and the proposed technique in Chirped-OFDM system

As is shown in Fig. 3, we can see that the same C^{η} only makes the tone reservation performance worse negligibly while it could reduce the calculation meaningfully through the structure of radix-2 FRFT algorithm.

5. Conclusion

A new PAPR reduction technique based on FRFT structure has been extended to the Chirped-OFDM system via tone reservation. For the symmetry and decimation in frequency of FRFT algorithm, the transform matrices on an intermediate stage are used to simplify the peak reduction signal. The derivation and simulation results show that its performance is close to conventional tone reservation with less computational complexity.

References

[1]. M. Martone, "A Multicarrier System Based on the Fractional Fourier Transform for Time-Frequency-Selective Channel", *IEEE trans. on Commun.*, vol. 49, no. 6, pp.1011-1020, Jun. 2001

[2]. T. Jiang, and Y. Wu, "An Overview Peak-to-Average Power Ratio Reduction Techniques for OFDM Signals", *IEEE trans. on Broadcasting*, vol. 54, no. 2, pp.257-268, Jun. 2008

[3]. J. Tellado and J. Cioffi. "Controlling clipping probability in DMT transmission", *ANSI Document, T1E1.4 Technique Subcommittee*, no.97-397, pp.1-7, Dec. 8,1997

[4]. Y. Ju and et al. "Analysis of Peak-to-average power ratio of a multi-carrier system based on the fractional Fourier transform". *IEEE Singapore International Conference on Communication System*. 2004, 165-168.

[5]. A. Ghassemi, and T. A. Gulliver, "Efficient Optimization for Tone Reservation OFDM", *ISITA*, 2008

[6]. Y. Ju and B. Barkat, "Analysis of Peak-to-average power ratio of a multicarrier system based on the fractional Fourier transform", *ICCS*, 2004